Coupling of Non–Axially Symmetric Hybrid Modes in Dielectric Resonators

KAWTHAR A. ZAKI, SENIOR MEMBER, IEEE, AND CHUNMING CHEN, STUDENT MEMBER, IEEE

Abstract —A method for the rigorous calculation of the coupling coefficient between dielectric resonators excited in non–axially symmetric hybrid modes is developed. A simplified model is derived which allows approximate accurate calculations of coupling, without solving the detailed boundary value problem. Experimental measurements of the coupling between the two lowest hybrid modes were performed to verify the models, and are found to be in excellent agreement with the calculations.

I. Introduction

Dielectric resonators made from highly temperature stable low-loss ceramics are finding increasing microwave applications due to their desirable properties and their commercial availability at reasonable prices. Miniaturization of components is a major driving factor in the use of these ceramics. Several applications (such as filters) require the knowledge of the coupling between resonators with high degree of accuracy.

Coupling between dielectric resonators excited in axially symmetric modes ($TE_{01\delta}$, TM_{010}, etc.) has been treated extensively in the literature [1]–[4]. Although a generalized approximate methodology that could be used for coupling computation between non–axially symmetric modes in dielectric resonators has been presented [5], there has been no rigorous treatment of the subject, with results that can directly be used for practical applications. The exception to this is the treatment for coupling of hybrid HE_{11} modes by iris presented in [6].

The model used in [1] to perform the coupling calculation between the resonators excited in $TE_{01\delta}$ mode approximates each of the resonators by an axial magnetic dipole. One of the dipoles radiates in the waveguide beyond cutoff. The fraction of the energy received by the other dipole is used as a measure of the coupling coefficient.

This paper presents a rigorous technique for coupling calculation between non–axially symmetric modes in cylindrical dielectric resonators enclosed in circular waveguides. The technique is based on solving the boundary value problem for the fields and resonant frequencies in the combined two-resonator structures. A circuit model representing the two coupled resonators is used to reduce the problem of coupling calculations to that of finding the resonant frequencies of two separate single resonators.

Section II describes the approach used for the solution and derives the relationships among various configurations of a single resonator's resonant frequencies and couplings. Field solutions using the mode-matching method are outlined in Section III. Section IV presents numerical and experimental results performed to verify the theory. A simplified approximate model is derived from the results of the rigorous analysis, which can be used for the design of the resonators without recourse to the detailed numerical solution of the boundary value problems. Experimental measurements performed to verify the models are presented and show excellent agreement with the theoretical calculations. Discussions and conclusions are contained in Section V.

II. Method of Coupling Calculation

Consider two identical circular cylindrical dielectric resonators of radius a and length t placed coaxially inside a perfectly conducting cylinder of radius b as shown in Fig. 1. The resonators have relative dielectric constant ϵ_r and are spaced a distance $2l$ apart. The planar end walls of the enclosure are perfectly conducting and are at a distance d each from the resonators ends.

This structure has several resonances which correspond to different field excitations. For axially symmetric modes which have no angular variation, the fields can be either transverse electric ($TE_{0m\delta}$) or transverse magnetic ($TM_{0m\delta}$). For fields which have angular variation, the modes on the structure will be hybrid modes possessing both electric and magnetic axial fields. For any particular mode near its resonance, an equivalent circuit for the two coupled resonators is shown in Fig. 2(a). This circuit consists of two series resonant circuits coupled by a mutual inductance M. The coupling coefficient k between the resonators is defined in terms of the equivalent circuit elements by

$$k = M/L. \quad (1)$$

An alternative form of the equivalent circuit is shown in Fig. 2(b). This form yields two-port parameters that are identical with those of the circuit of Fig. 2(a), but it is more convenient to use in the following discussions. If the symmetry plane in Fig. 2(b) is replaced by a short circuit, the resulting circuit will have a resonant frequency f_e which is equal to the resonant frequency of the single

Manuscript received April 3, 1987; revised July 16, 1987. This work was supported in part by the National Science Foundation under grant ECS 8320249.

The authors are with the Electrical Engineering Department, University of Maryland, College Park, MD 20742.

IEEE Log Number 8717111.

0018-9480/87/1200-1136$01.00 ©1987 IEEE

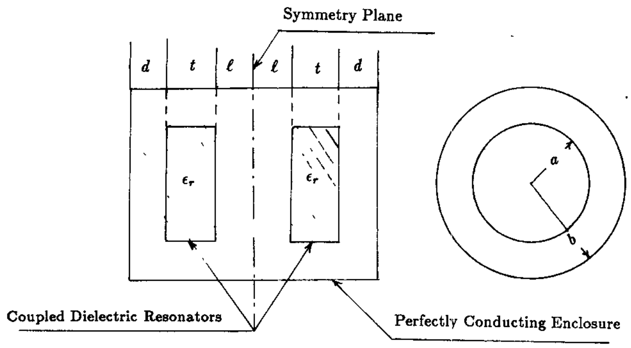

Fig. 1. Two coupled dielectric resonators in a perfectly conducting cylindrical enclosure.

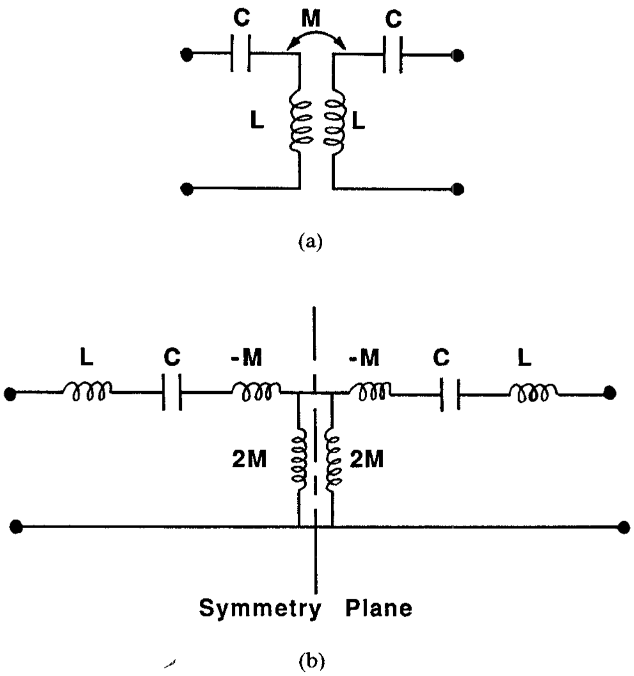

Fig. 2. Two forms of the equivalent circuit of the coupled resonators.

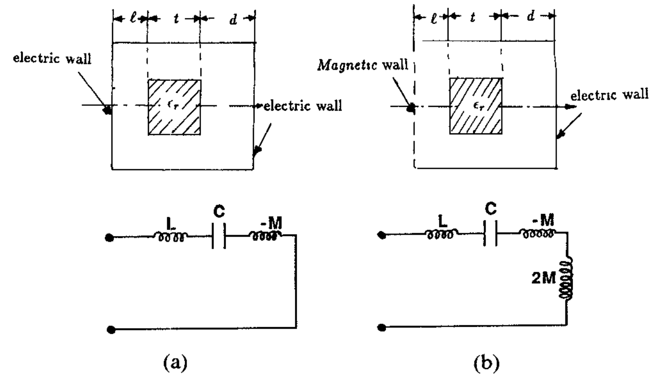

Fig. 3. Equivalent circuits with symmetry planes replaced by (a) electric wall and (b) magnetic wall.

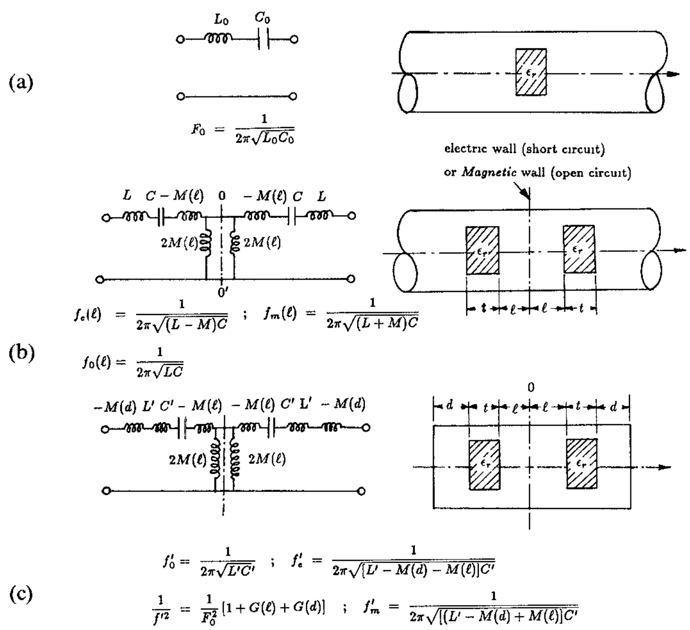

Fig. 4. (a) Equivalent circuit of a single resonator in an infinite guide. (b) Two coupled resonators in an infinite guide. (c) Two coupled resonators in a guide shortened at both ends.

resonator shown in Fig. 3(a). This structure is obtained from Fig. 1 by placing an electric wall (hence the suffix e in f_e) in the symmetry plane. It is easily seen that the value of f_e is related to the circuit parameters by

$$f_e = \frac{1}{2\pi\sqrt{(L-M)C}}. \tag{2}$$

Similarly, replacing the symmetry plane in Fig. 2(b) by an open circuit results in a single resonant circuit having a resonant frequency f_m which is equal to the resonant frequency of the structure shown in Fig. 3(b), having magnetic wall in the symmetry plane of Fig. 1. The value of f_m is given by

$$f_m = \frac{1}{2\pi\sqrt{(L+M)C}}. \tag{3}$$

Equations (1)–(3) can be solved for the coupling coefficient k:

$$k = \frac{M}{L} = \frac{f_e^2 - f_m^2}{f_e^2 + f_m^2}. \tag{4}$$

Calculation of f_e and f_m as a function of the physical parameters of the structure is obtained by solving the boundary value problem as outlined in Section III below.

For dielectric resonators of given dimensions, the resonant frequencies f_e and f_m can be considered functions of the two variables l and d. It is clear that the coupling coefficient k will be strongly dependent on l, and to a lesser extent on d. To explore the nature of this dependence and to develop a model for the coupling in terms of one variable (i.e., l or d), consider the cases shown in Fig. 4. A single resonator placed in an infinite guide, as shown in Fig. 4(a), will have resonant frequency F_0. When two such resonators are placed in the infinite waveguide and spaced $2l$ apart, as shown in Fig. 4(b), the resonant frequency $f_0(l)$ of each of the two resonators changes for two reasons:

(i) the effect of the other higher order modes;
(ii) the effect of the coupling between the resonators.

The first effect is reflected in the equivalent circuit in Fig. 4(b), showing LC different from L_0C_0 of Fig. 4(a), and the second effect is accounted for by the T section composed of series arms of $-M$ and the two shunt branches $2M$. The circuit in Fig. 4(b) is characterized completely for any l by the frequencies $f_e(l)$ and $f_m(l)$ obtained by placing an electric wall (short circuit) and magnetic wall (open circuit) in the symmetry plane 0–0′, respectively. It is easy to show that the following relations hold:

$$\frac{1}{f_0^2(l)} = \frac{1}{2}\left[\frac{1}{f_e^2(l)} + \frac{1}{f_m^2(l)}\right] \tag{5}$$

$$\lim_{l\to\infty} f_0(l) = \lim_{l\to\infty} f_m(l) = \lim_{l\to\infty} f_e(l) = F_0. \tag{6}$$

Let the functions $G(l)$ and $H(l)$ be defined by

$$G(l) = \frac{F_0^2}{f_0^2(l)} - 1 \tag{7}$$

$$H(l) = \frac{F_0^2}{2}\left[\frac{1}{f_m^2(l)} - \frac{1}{f_e^2(l)}\right]. \tag{8}$$

The functions $G(l)$ and $H(l)$ have a physical interpretation associated with them. In terms of the equivalent circuits of Fig. 4, the function G is simply the relative change in the product LC of the value L_0C_0, while the function H is the ratio MC/L_0C_0 for two resonators in the infinite waveguide. The parameters of the completely enclosed resonator structure shown in Fig. 4(c) can be easily computed from a knowledge of F_0 and the functions $G(l)$ and $H(l)$ by the following relationships:

Equivalent isolated resonator resonant frequency:

$$f_0' = \frac{1}{2\pi\sqrt{L'C'}} \tag{9a}$$

$$\frac{1}{f_0'^2} = \frac{1}{F_0^2}[1 + G(l) + G(d)]. \tag{9b}$$

Resonant frequency with electric wall in the symmetry plane 0–0′ (Fig. 4(c)):

$$f_e' = \frac{1}{2\pi\sqrt{[L' - M(d) - M(l)]C'}} \tag{10a}$$

$$\frac{1}{f_e'^2} = \frac{1}{f_0'^2} - \frac{1}{F_0^2}[H(d) + H(l)]. \tag{10b}$$

Resonant frequency with magnetic wall in the symmetry plane 0–0′ (Fig. 4(c)):

$$f_m' = \frac{1}{2\pi\sqrt{L' - M(d) + M(l)]C'}} \tag{11a}$$

$$\frac{1}{f_m'^2} = \frac{1}{f_0'^2} - \frac{1}{F_0^2}[H(d) - H(l)]. \tag{11b}$$

Coupling coefficient between the two resonators:

$$k'(l,d) = \frac{f_e'^2 - f_m'^2}{f_e'^2 + f_m'^2} = \frac{H(l)}{1 + G(l) + G(d) - H(d)}. \tag{12}$$

To summarize, all parameters of the completely enclosed resonator of Fig. 4(c) can be determined from the resonant frequency F_0 of a single isolated resonator in an infinite waveguide and the functions $G(l)$, $H(l)$ defined in (7) and (8), respectively.

III. Field Solutions Using Mode Matching

This section briefly summarizes the method of solution for the resonator's fields and resonant frequencies. To calculate the frequencies f_0, f_e, and f_m, defined in the last section (see Fig. 4(a) and 4(b)), the method of mode matching [7] is used. In this method the resonator is considered to consist of cascaded sections of a circular empty waveguide and a dielectric loaded waveguide. The transverse fields in the dielectric loaded region are represented as a linear combination of incident and reflected hybrid modes of an infinite dielectric-loaded-waveguide. The transverse fields in the empty guides are represented as a linear combination of normal waveguide modes (TE_{mn} and TM_{mn} modes) satisfying the boundary condition at the end plane of vanishing tangential electric or magnetic fields for an electric wall or magnetic wall, respectively. For the case of the resonator in Fig. 4(a), the transverse fields in the infinite region of the empty waveguide are represented as a combination of traveling (or evanescent) normal waveguide modes. All the fields must have the same angular variation ($\sin m\phi$ and $\cos m\phi$). The boundary conditions to be satisfied are that the tangential electric and magnetic fields be continuous. An integrated weighted mean square error criterion is used to convert the equations resulting from the application of these boundary conditions to an infinite set of linear homogeneous equations with the unknowns as the mode expansion coefficients. Numerical solution of this system is accomplished by truncating it to a finite size N. The roots of the equations resulting from equating the determinant of this matrix to zero are the frequencies F_0, f_e, and f_m, as appropriate.

This procedure was implemented numerically by adopting the computer programs developed in [7]. The programs were thoroughly tested and convergence criteria similar to those described in [7] were used to ascertain convergence.

IV. Numerical and Experimental Results

Typical results of the resonant frequencies for different modes are shown in Fig. 5. Parameters used in the generation of this figure are $\epsilon_r = 35.74$, $a = 0.34$ in, $b = 0.5$ in, $t = 0.3$ in and $d = 0.2$ in. Also shown in Fig. 5 are measured resonances of the structure composed of two such resonators versus the half separation between the resonators l. The measurements were made using a structure of two dielectric resonators in cylindrical enclosures, with a coaxial SMA connector lightly coupled to one of the resonators. The spacing between the resonators was varied by inserting thin spacers. The probe depth was changed for each spacing so that the reflection coefficient at resonance was at least 20 dB. Frequencies of minimum reflection were measured accurately for each mode. For any particu-

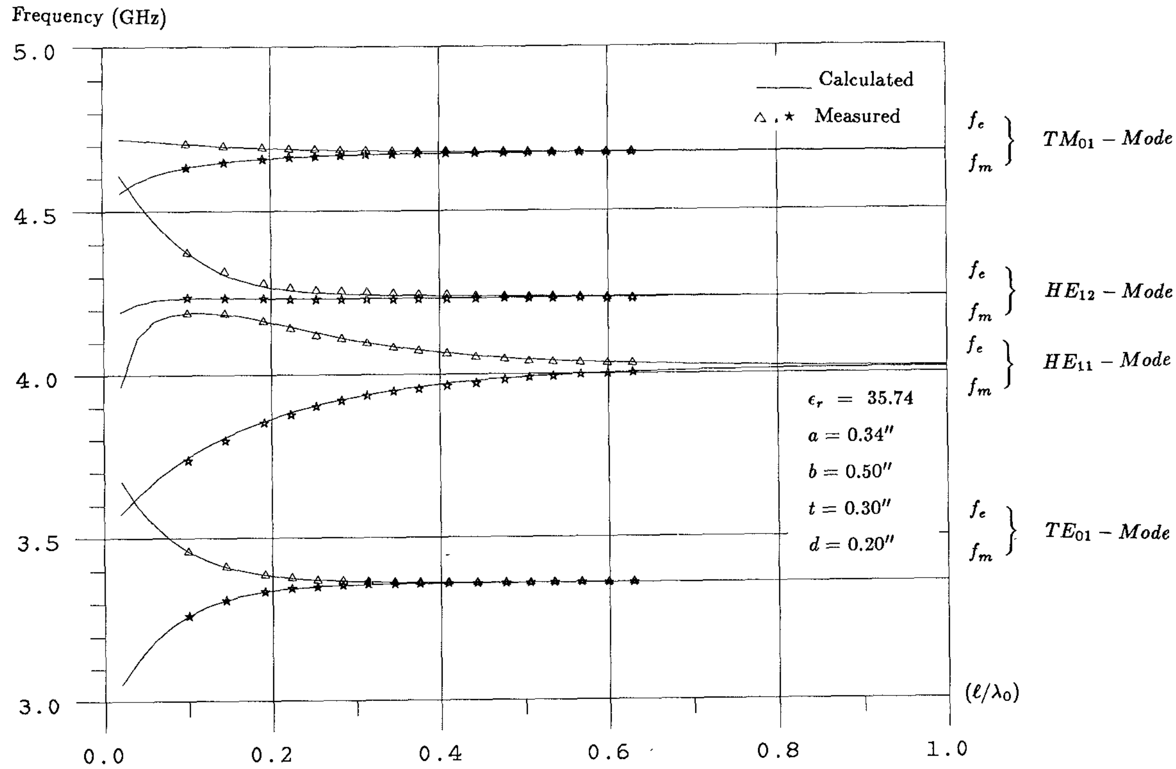

Fig. 5. Calculated and measured results for f_e and f_m of the two lowest hybrid modes.

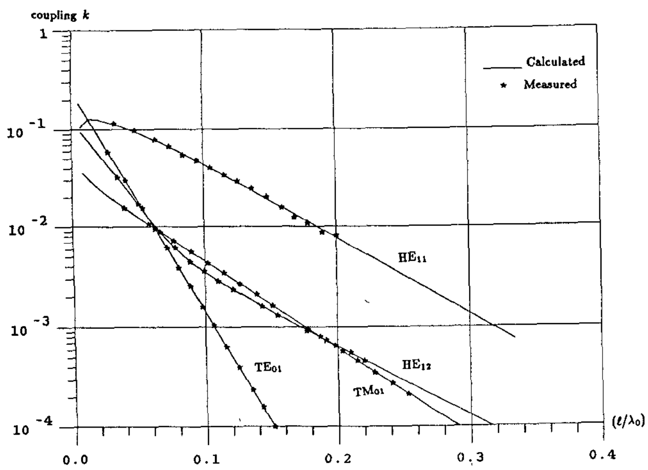

Fig. 6. Calculated and measured coupling k between two resonators for the two lowest hybrid modes. (Resonator parameters are the same as in Fig. 5.)

lar mode, the lower frequency is identified as f_m, while the higher frequency is identified as f_e. Agreement between both the theory and the measured data is quite good. For the same parameters given above, the measured and computed coupling coefficients k as a function of l calculated from (4) are shown in Fig. 6.

To verify the model presented in Section II for calculation of f_e and f_m in the case of an enclosed resonator from the values in an infinite guide, the same resonator parameters were used to calculate F_0, the functions G and H, defined in (7) and (8), and the variation of f_e and f_m with l, for d as a parameter. All frequency data were normalized to $F_0 = 3.9439$ GHz, and all lengths are normalized to free-space wavelength $\lambda_0 = c/F_0$. Fig. 7(a) gives the variation of f_e and f_m with (l/λ_0) for the infinite waveguide case $(d/\lambda_0 = \infty)$. The functions G and H are shown in Fig. 7(b) and (c), respectively. The variations of (f_e/F_0) and (f_m/F_0) with (l/λ_0) for (d/λ_0) as a parameter are shown in Figure 7(d). In this figure the solid lines give the exact values computed from the direct numerical solution of the boundary value problem. The dashed lines are the values calculated from the circuit model described in Section II. For large values of (d/λ_0), the two solutions are identical. As (d/λ_0) decreases, the approximate solution for (f_e/F_0) starts to deviate from the exact solution, particularly for small values for (l/λ_0). For $(d/\lambda_0) > 0.1$, the error in the approximate solution for (f_e/F_0) is less than 1 percent, while (f_m/F_0) is accurate with error less than 0.2 percent for all cases. The error in the approximate model for small values of (d/λ_0) is due to the fact that this model does not include any higher order mode interactions between the short circuits at the two ends of the resonators. This assumption fails for small values of (d/λ_0) and (l/λ_0).

Coupling was computed for a wide range of parameters of the resonators, the enclosure and the spacing $2l$ between the resonators, for the lowest frequency hybrid mode (HE_{11}). A few results of these calculations are summarized in Fig. 8. From Fig. 8, it is seen that for values of the coupling k less than 0.075, the coupling can be accurately described by the expression

$$k = k_0 e^{-2\alpha l} \tag{13}$$

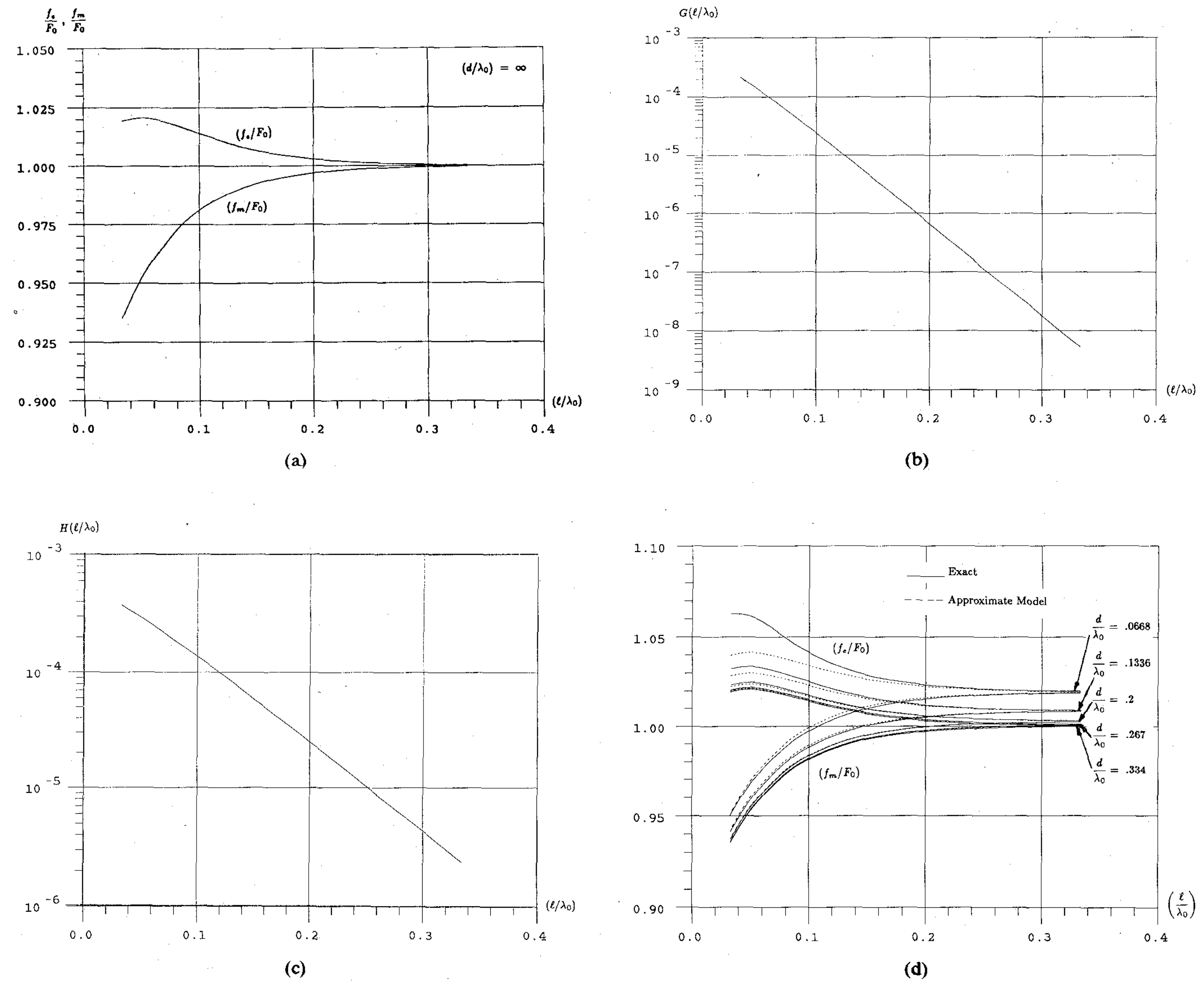

Fig. 7. (a) Variation of f_e/F_0 and f_m/F_0 with l/λ_0 for an infinite guide $(d/\lambda_0)=\infty$. (b) Variation of the function $G(l/\lambda_0)$ with l/λ_0. (c) Variation of the function $H(l/\lambda_0)$ with l/λ_0. (d) Variation of f_e/F_0 with l/λ_0 for different d/λ_0.

where α is the attenuation constant of the TE_{11} mode in a circular waveguide of radius b at the resonant frequency of the resonator and k_0 is a constant that depends on the resonator parameters. Table I gives a typical range of parameters for two coupled resonators. Values of α and k_0 determined from numerical computations are shown. The table also compares the attenuation factors α as determined from the least square fit of the computed points and from the waveguide attenuation constant $\alpha_{\text{W.G.}}$, given by

$$\alpha_{\text{W.G.}}=\left(\frac{1.841}{b}\right)\sqrt{1-\left(\frac{2\pi bF_0}{1.841c}\right)^2} \tag{14}$$

where c is the speed of light and F_0 is the resonant frequency of a single resonator in an infinite waveguide. Agreement between the values of $\alpha_{\text{W.G.}}$ and α is excellent (better than 0.06 percent).

The constant k_0 is a complicated function of the structure parameters. Attempts were made to obtain empirical formulas which relate k_0 to the physical dimensions of the structure. The best form obtained for such a formula is

$$k_0=c_0(a/b)^{c_1}(t/\lambda_0)^{c_2}(d/\lambda_0)^{c_3}\epsilon_r^{c_4} \tag{15}$$

where λ_0 is the free-space wavelength at F_0. Numerical fit of the data obtained for k_0 for a wide range of parameters yields the following values for the coefficients in Eq. (15):

$$c_0=1.3293 \qquad c_1=1.1057 \qquad c_2=-0.6298$$
$$c_3=-0.2154 \qquad c_4=0.2394. \tag{16}$$

These values yield k_0 with an accuracy better than 10 percent for frequencies F_0 in the range of 1 GHz to 10 GHz and ϵ_r in the range 30 to 90.

One way to adjust the coupling coefficient between two resonators without changing their spacing $(2l)$ is to insert two thin conducting obstacles (e.g. screws) opposite to

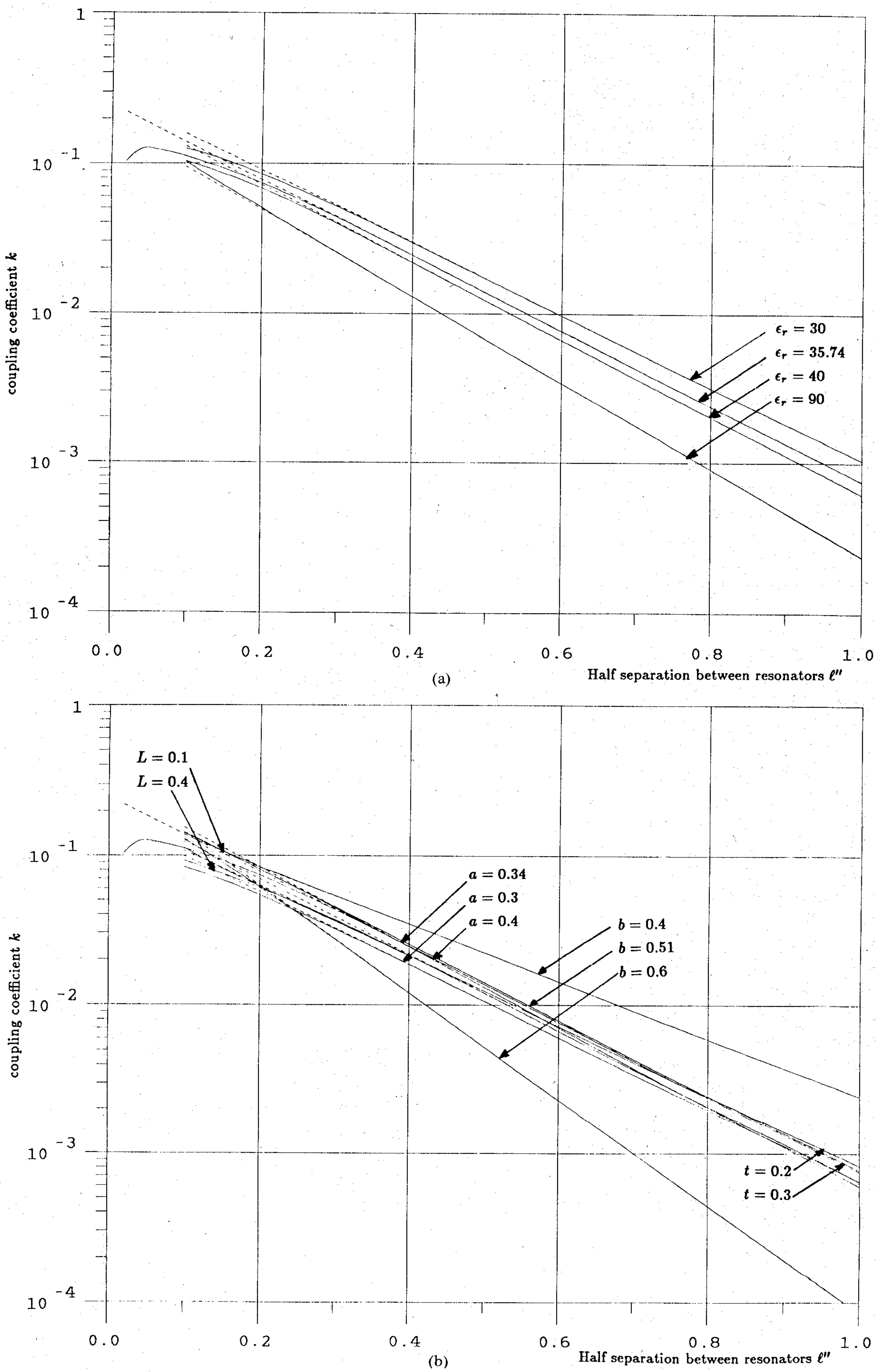

Fig. 8. (a) Variation of coupling coefficient with separation between resonators for various parameters. (b) Variation of coupling coefficient with separation between resonators for various parameters.

TABLE I
TYPICAL PARAMETERS FOR COUPLED RESONATORS

a''	b''	t''	ϵ_r	d''	f_0	α	$\alpha_{W.G.}$	k_0
.30	.51	.3	35.74	.2	4.2807	2.8013	2.7995	0.17493
.34	.4	.3	35.74	.2	3.8133	4.1321	4.1313	0.32940
.34	.6	.3	35.74	.2	4.0132	2.2043	2.2042	0.20129
.34	.51	.3	35.74	.2	4.0180	2.9011	2.9011	0.24937
.34	.51	.2	35.74	.2	4.4755	2.7138	2.7125	0.18709
.34	.51	.3	30.0	.2	4.2932	2.7959	2.7951	0.27852
.34	.51	.3	40.0	.2	3.8373	2.9776	2.9771	0.23798
.34	.51	.3	90.0	.2	2.6437	3.3252	3.3249	0.18575
.34	.51	.2	35.74	.1	4.0174	2.9096	2.9096	0.25895
.34	.51	.3	35.74	.4	3.9754	2.9259	2.9254	0.22472
.4	.51	.3	35.74	.2	3.5166	4.1318	4.1321	0.28839

each other midway between the resonators [8]. It is interesting to note that the effect of such an obstacle is to increase the coupling between the resonators as their penetration is increased. This is observed experimentally, and can also be explained physically in terms of the model used for coupling computation (eq. (4)). The conducting obstacles do not affect f_e since the tangential electric field is zero in the plane of the obstacles. On the other hand, for f_m, a tangential electric field exists in the plane of the obstacles; hence the value of f_m is lowered as the penetration of the obstacles is increased. Thus, from (4) it becomes apparent that the coupling k_0 actually increases as the depth of penetration of the obstacle is increased.

V. CONCLUSIONS

The coupling between hybrid modes in dielectric resonators can be accurately calculated by solving for the resonant frequencies of single resonators with electric and magnetic walls. Experimental results verified the accuracy of the calculations. An approximate circuit model is derived which uses the calculated resonant frequencies of resonators in infinite waveguide to derive the coupling parameters for any dimensions of the enclosure. This approximate model provides excellent accuracy for a wide range of parameters of practical interest. A simplified exponential model that requires only the two parameters k_0 and α is postulated which accurately predicts the results for a limited, but wide range of parameters. The attenuation α is simply the attenuation constant of the TE_{11} mode in a waveguide beyond cutoff of the same radius as the enclosure, at the resonant frequency of the resonator. An empirical formula is given for the constant k_0 as a function of the resonator parameters.

REFERENCES

[1] S. B. Cohn, "Microwave bandpass filters containing high-Q dielectric resonators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, pp. 218–227, Apr. 1968.

[2] Y. Kobayashi and H. Furukawa, "Elliptic bandpass filters using four TM_{010} dielectric rod resonators," in *IEEE MTT-S 1986 Int. Microwave Symp. Dig.*, June 1986, pp. 353–356.

[3] P. Guillon, M. P. Chong, and Y. Garault, "Dielectric resonators band pass filter with high attenuation rate," in *IEEE MTT-S 1984 Int. Microwave Symp. Dig.*, pp. 240–242.

[4] J. K. Plourde and C. L. Ren, "Application of dielectric resonators in microwave components," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-29, pp. 754–770, Aug. 1981.

[5] J. Van Bladel, "The excitation of dielectric resonators of very high permittivity," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-23, pp. 208–217, Feb. 1975.

[6] S. J. Fiedziusko, "Dual-mode dielectric resonator loaded cavity filters," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 1311–1316, Sept. 1982.

[7] K. A. Zaki and C. Chen, "New results in dielectric loaded resonators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, pp. 815–824, July 1986.

[8] K. A. Zaki, C. Chen, and A. E. Atia, "Canonical and longitudinal dual mode dielectric resonator filters without iris," *IEEE Trans. Microwave Theory Tech.*, pp. 1130–1135, this issue.

Kawthar A. Zaki (SM'85) received the B.S. degree (with honors) from Ain Shams University, Cairo, Egypt, in 1962, and the M.S. and Ph.D. degrees from the University of California, Berkeley, in 1966 and 1969, respectively, all in electrical engineering.

From 1962 to 1964, she was a Lecturer in the Department of Electrical Engineering, Ain Shams University. From 1965 to 1969, she held the position of Research Assistant in the Electronics Research Laboratory, University of California, Berkeley. She joined the Electrical Engineering Department, University of Maryland, College Park, in 1970, where she is presently an Associate Professor. Her research interests are in the areas of electromagnetics, microwave circuits, optimization, computer-aided design, and numerical techniques.

Dr. Zaki is a member of Tau Beta Pi.

Chunming Chen (S'85) was born in Taiwan, Republic of China, in 1958. He received the B.S. degree from the National Tsing Hua University, Taiwan, in 1981 and the M.S. degree from the University of Maryland, College Park, in 1985, both in electrical engineering.

Since 1984, he has worked as a Research Assistant in the Department of Electrical Engineering, University of Maryland, College Park. He is now working towards the Ph.D. degree in the area of microwave components and circuits.